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"Modern Physics"

What is it?

See Wikipedia "Modern Physics"

- Everything after Newton's & Maxwell's theories

1642-1727

1831-1879

1900s +

Modern physics explains things that Newton & Maxwell could not. Modern Physics includes relativity and quantum mechanics

"Relativity"

What is it?

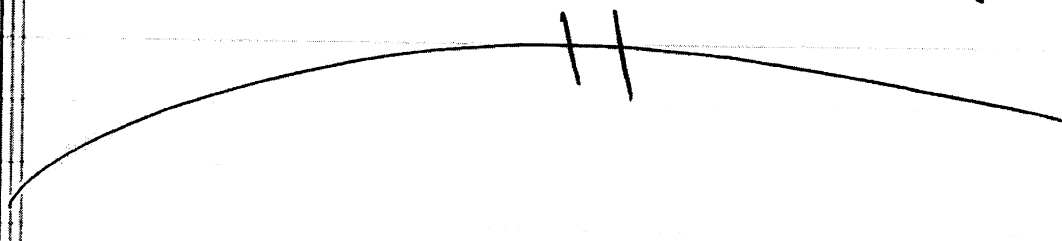
covered topic

→ Special Relativity: Very fast objects, no external forces
General Relativity: Very fast objects, with external forces
Very massive, dense objects
(black holes)

covered topic

→ "Quantum Mechanics": The physics of really small and light things

"Relativistic Quantum Mechanics": The physics of really, small, fast things.



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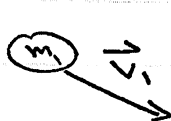
Review of 2-body Collision Kinematics (Review)

What is it?

Newtonian Physics

Bang two things together, and two things separate after the collision

$t=0$

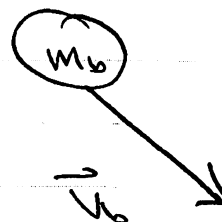
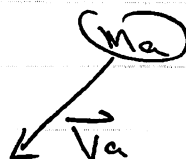


$t=t_1 > 0$



} figure 1

$t=t_2 > t_1$



Simplification / Specialization

- 1) two bodies in, two bodies out
- 2) bodies are not rotating, before or after

Why do we study this?

NE applications: neutron-neutron scattering
neutron-proton scattering (neutron/wall)

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neutron deuteron scattering (CANDU reactor)
 proton/alpha/electron-nucleus scattering
 (Rutherford scattering)
 neutron-nucleus scattering
 (fuel bundles, containment)
 neutron shielding
 fission
 ...

← got to here
 9/9/09

How do we analyze this?

(CoE) Conservation of Energy (Kinetic + other)

(CoM) Conservation of momentum

(CoMa) Conservation of mass

Review If object, mass m , has velocity $\vec{v}$,

its kinetic energy $K = \frac{1}{2} m \vec{v} \cdot \vec{v} = \frac{1}{2} m v^2$ (a scalar)
 $v^2 = \vec{v} \cdot \vec{v}$ shorthand

its momentum is $\vec{p} = m \vec{v}$ (a 3-vector)

Applying the conservation laws to Figure 1...

before

after

$$\text{CoM} \Rightarrow \vec{p}_1 + \vec{p}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_a \vec{v}_a + m_b \vec{v}_b = \vec{p}_a + \vec{p}_b \quad (1)$$

$$\text{CoE} \Rightarrow K_1 + K_2 + Q = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + Q = \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_b v_b^2 = K_a + K_b \quad (2)$$

$$\text{CoM} \Rightarrow m_1 + m_2 = m_a + m_b \quad (3)$$

(4)

What is Q?

Q is the reaction Q-value. See K2 p. 380-381

 $Q > 0$ "exothermic" "exoergic" energy given off
(explosions)

 $Q < 0$ "endothermic" "endoergic" energy required
to make work
(sticky collisions)

 $Q = 0$ "elastic"
Complexity
 ① ② ③ have, in principle 17 variables,
potentially 17 unknowns

$$m_1, m_2, m_a, m_b, \vec{v}_1, \vec{v}_2, \vec{v}_a, \vec{v}_b, \vec{Q}$$

If we know 14 of them, we can solve for the other 3

If we know 13 " " , we can relate 2 of them

" " " 12 " " , " " " one to 2 others

and so on

 Fortunately, most cases simplify greatly.
We'll get to that soon.

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Notes: If only one mass is unknown, we can relate it to the other three.

e.g. $m_b = m_1 + m_2 - m_a$ (4)

Notes, we can easily eliminate one of the v 's by simple math

e.g. eliminating $v_b \dots$

reorganize.

reorg (1) $\Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 - m_a \vec{v}_a = m_b \vec{v}_b$ (5)

$2m_b$ (2) $\Rightarrow m_b (m_1 v_1^2 + m_2 v_2^2 - m_a v_a^2 + Q) = m_b^2 v_b^2$ (6)

(5)² - (6) $\Rightarrow$

$$|m_1 \vec{v}_1 + m_2 \vec{v}_2 - m_a \vec{v}_a|^2 - \underbrace{(m_1 + m_2 - m_a)}_{=m_b} (m_1 v_1^2 + m_2 v_2^2 - m_a v_a^2 + Q)$$

$= 0$
(9)

"b" no longer appears

Fortunately, most interesting cases are simpler.



Example 1

$$m_1 = m_2 = m_a = m_b = m$$

$$Q = 0$$
$$\vec{V}_2 = 0$$

(billiard balls)

neutron-neutron scatter
elastic

$$\text{CoE} \Rightarrow \frac{1}{2} m V_1^2 = \frac{1}{2} m V_a^2 + \frac{1}{2} m V_b^2$$

(10)

$$\text{CoM} \Rightarrow m \vec{V}_1 = m \vec{V}_a + m \vec{V}_b$$

(11)

Note that all the "m"s cancel out, so, we can also write

$$V_1^2 = V_a^2 + V_b^2$$

(12)

$$\vec{V}_1 = \vec{V}_a + \vec{V}_b$$

(13)

Q: What is the angle between particles a and b?

A: (13)² - (12) $\Rightarrow$

$$|\vec{V}_a + \vec{V}_b|^2 - (V_a^2 + V_b^2) = \underbrace{|\vec{V}_1|^2 - V_1^2}_{=0}$$

$$\cancel{V_a^2} + 2\vec{V}_a \cdot \vec{V}_b + \cancel{V_b^2} - (\cancel{V_a^2} + \cancel{V_b^2}) = 0$$

$$\Rightarrow \vec{V}_a \cdot \vec{V}_b = 0 \quad (14)$$

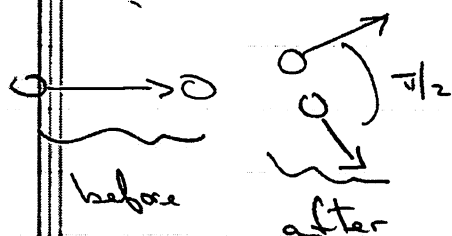
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Let's rewrite as follows

$$\vec{V}_a \cdot \vec{V}_b = V_a V_b \cos \Theta_{ab} = 0 \quad (15)$$

Conclusions:

1) if $V_a \neq 0$ & $V_b \neq 0$, $\cos \Theta_{ab} = 0$



$\Rightarrow$ the angle between the outgoing particles is $\frac{\pi}{2}$, 90° , $\perp$, a right angle

2) if V_a or V_b is 0, Θ_{ab} is undefined.

If 2) is the case, let's let $\vec{V}_b = 0$

In this case (13) $\Rightarrow \vec{V}_a = \vec{V}_i$



If both $\vec{V}_a \hat{=} \vec{V}_b$ are $\vec{0}$

$$(13) \Rightarrow \vec{V}_i = 0$$

oo

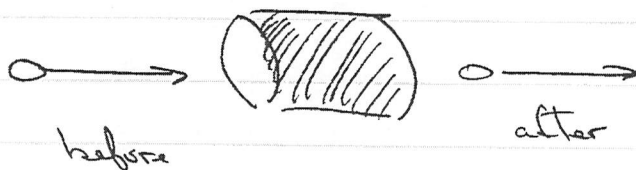
two billiard balls at rest.

Not very interesting, but it is covered by the math.

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The math is even more general than that.

Imagine that the collision is hidden, and you observe



The only things you could say:

- 1) The same ball went out as went in
- 2) A different ball of identical mass came out
- 3) anything else?

⋮

Kinematics is a gross but powerful simplification. It does not say anything about how the particles scattered. That's a much more complicated discussion. However, the conservation laws still apply, and they allow us to say many important things about these collisions.

And, these kinematical relations hold for submicroscopic particles, as well as billiard balls, asteroids, planets and galaxies.

Example 1a (Like Example 1, but unequal masses) $Q=0$ projectile, mass m , velocity $\vec{v}_0$ target at rest, mass M . No mass exchange.

Where can the projectile scatter to?

(What angles?)

$$\text{CoE: } \frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} M V^2$$

$$\text{or, } v_0^2 = v^2 + n V^2 \quad (1) \quad \text{where } n = \frac{M}{m}$$

 $n > 1$ target heavier $n < 1$ target lighter

$$\text{CoM } m \vec{v}_0 = m \vec{v} + M \vec{V}$$

or

$$\vec{v}_0 = \vec{v} + n \vec{V} \quad (2)$$

In order to compute the scattering angle, θ , of the projectile, eliminate V .Put V on one side of the equations

$$\therefore (1) \Rightarrow v_0^2 - v^2 = n V^2 \quad (3)$$

$$(2) \Rightarrow \vec{v}_0 - \vec{v} = n \vec{V} \quad (4)$$

$$\therefore n(3) - |4|^2 \Rightarrow$$

$$n(v_0^2 - v^2) - (v_0^2 - 2\vec{v}_0 \cdot \vec{v} + v^2) = 0$$

 V is eliminated



$$\vec{u}_0 \cdot \vec{u} = u_0 u \cos \theta \quad \leftarrow \text{scattering angle}$$

(10)

reorganizing

$$(n+1)u^2 - 2u_0 u \cos \theta - (n-1)u_0^2 = 0 \quad (5)$$

Solving the quadratic eqⁿ

$$ax^2 + 2bx + c = 0 \Rightarrow x = -b \pm \sqrt{b^2 - ac}$$

$$\text{So, } u = \frac{u_0 \cos \theta \pm \sqrt{u_0^2 \cos^2 \theta + (n^2 - 1)u_0^2}}{(n+1)}$$

or,

$$\frac{u}{u_0} = \frac{\cos \theta \pm \sqrt{n^2 - \sin^2 \theta}}{(n+1)} \quad (6a)$$

equivalent form $\left\{ \frac{u}{u_0} = \frac{\cos \theta \pm \sqrt{n^2 - 1 + \cos^2 \theta}}{(n+1)} \right. \quad (6b)$

Case 1: $n > 1$ heavier target

$\sqrt{\quad}$ is always real

$\sqrt{\quad} > |\cos \theta|$, so only the +ve sign

$$n > 1; \quad \frac{u}{u_0} = \frac{\cos \theta + \sqrt{n^2 - 1 + \cos^2 \theta}}{(n+1)} \quad (7)$$

$\cos \theta$ can be anything

$-1 \leq \cos \theta \leq 1$ any direction

Case 2: $n < 1$; lighter target

1) In order for $\sqrt{\quad}$ to be real

$$\sin \theta \leq n$$

i.e. $0 \leq \theta \leq \theta_{\max}$ where $\sin \theta_{\max} = n$

or $\pi - \theta_{\max} \leq \theta \leq \pi$

2) In order for v to be true (really, $v = |\vec{v}|$)

$$\theta \leq \pi$$

3) $\sqrt{\quad} < \cos \theta$ $\therefore$ both signs

$$\frac{v}{v_0} = \frac{\cos \theta \pm \sqrt{n^2 - \sin^2 \theta}}{1 + n} \quad \begin{array}{l} \theta < \theta_{\max} \\ \sin \theta_{\max} = n \end{array}$$

Conclusions If $M > m$, projectile can end up anywhere $0 \leq \theta \leq \pi$

If $m > M$, $\theta \leq \theta_{\max}$, $\sin \theta_{\max} = \frac{M}{m}$

$\therefore$ each angle has two possible velocities

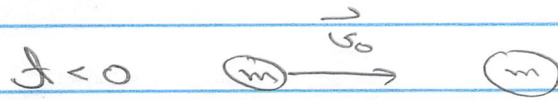
Stuff to do on your own:

1) Find the relationship between M' , speed, V , and its scattering angle.

Example 2

Same as Example 1 (billiard balls), but this time, the projectile is target stick.

Visually,



By $\vec{CoM}$, m and $2m$ must move along the same line. So, it's a 1D problem

$$CoE \Rightarrow \frac{1}{2} m v_0^2 = \frac{1}{2} (2m) V^2 \quad (1)$$

$$CoM \Rightarrow m v_0 = 2m V \quad (2)$$

$$\frac{1}{2} m \times (1) \Rightarrow v_0^2 = 2V^2 \quad (3)$$

$$\left(\frac{1}{2} m \times (2)\right)^2 \Rightarrow v_0^2 = 4V^2 \quad (4)$$

Inconsistent. Must include Q

Retry with Q:

$$\text{CoE} \Rightarrow \frac{1}{2}mv_0^2 + Q = \frac{1}{2}(2m)V^2 \quad (5)$$

$$\text{CoM} \Rightarrow mv_0 = 2mV \quad (6)$$

From (6) $V = \frac{v_0}{2}$, substitute this in (5) to find Q

$$\therefore \frac{1}{2}mv_0^2 + Q = m\left(\frac{v_0}{2}\right)^2$$

$$\boxed{Q = -\frac{1}{4}mv_0^2} \quad (7)$$

$$\text{or} \quad \boxed{Q = -\frac{1}{2}K_0} \quad (7')$$

Therefore, in this situation

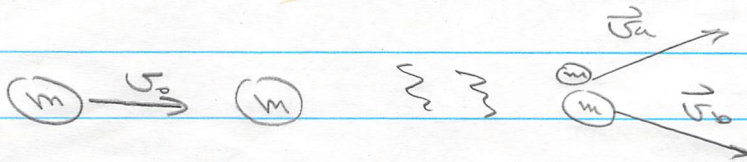
$\frac{1}{2}$ of K_i goes into Q (heat)

$\frac{1}{2}$ of K_i goes into the K of the fused particles

Q is negative (endo thermic)

Example 3

Like examples 1 & 2 except $-\frac{K_0}{2} \leq Q \leq 0$



$$\text{CoE: } \frac{1}{2}m\vec{v}_0^2 + Q = \frac{1}{2}m\vec{v}_a^2 + \frac{1}{2}m\vec{v}_b^2 \quad (1)$$

$$\text{CoM } m\vec{v}_0 = m\vec{v}_a + m\vec{v}_b \quad (2)$$

Special case

$$|\vec{v}_a| = |\vec{v}_b| = v \quad (\text{same magnitude, different dir'n})$$

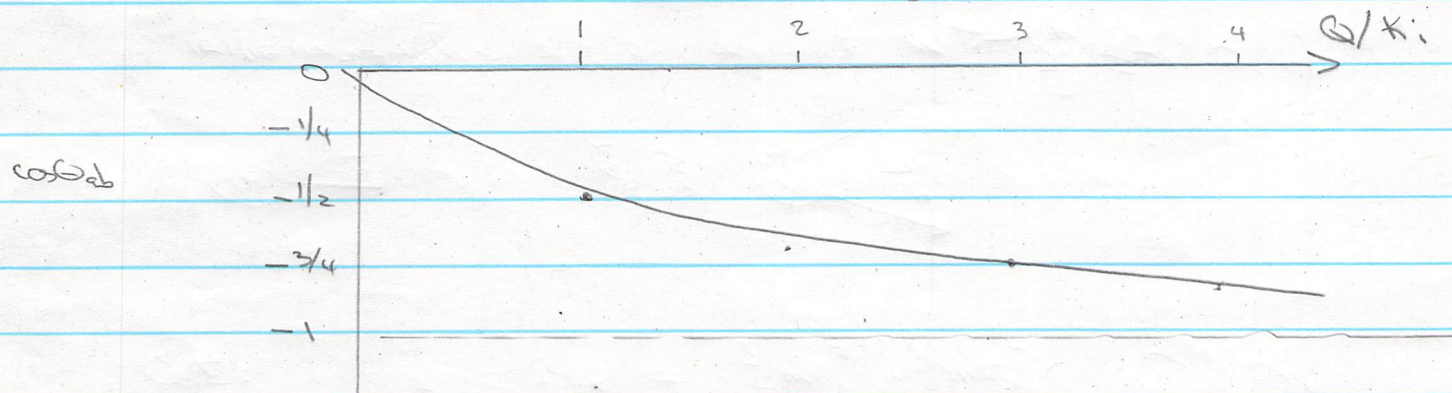
In this case: (This may be an assigned problem)

$$\boxed{\begin{array}{l} \cos \theta_{ab} = \frac{-Q}{K_0 + Q} \\ \text{angle} \\ \text{between} \\ \vec{v}_a \text{ and } \vec{v}_b \end{array}} \quad (3)$$

Discuss Does this make sense?

$$Q = 0 \quad \cos \theta_{ab} = 0 \quad (\text{if } K_0 \geq 0) \\ (\text{result of example 1})$$

$$Q = -K_0/2 \quad \cos \theta_{ab} = \frac{K_0/2}{K_0/2} = 1 \\ (\text{result of example 2})$$

$Q > 0?$ $\cos \theta_{ab}$ is negative

Q Explain what this means

$\frac{Q}{K} \rightarrow \infty$ (big explosion)

$\cos \theta_{ab} \rightarrow -1$ (pieces go in opposite directions)